Vanishing Fermi to Gamow-Teller Mixing Ratio of the β^+ Decay of ^{58}Co

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Z. Naturforsch. **45a**, 107–109 (1990); received November 6, 1989

Since 1956, fourteen experimentally-deduced values of the Fermi to Gamow-Teller mixing ratio $y = C_v M_F / C_A M_{GT}$ were published, varying from 0.05 to −0.36 with most values consistent with $y = 0$, particularly those from recent measurements. Our calculation, using the Nilsson model, yields $y = 2.07 \times 10^{-4}$, which can be taken as zero. Our result is therefore consistent with time-reversal invariance. Furthermore, the vanishing value of y arises both from the ΔT selection rule and the ΔK selection rule.

Introduction

The Fermi to Gamow-Teller mixing ratios, $y = C_v M_F / C_A M_{GT}$, in isospin-forbidden beta decays ($J \neq 0$, $\Delta J = 0$, $\Delta T = \pm 1$ and no parity change) have been extensively studied [1] because of their fundamental significance. The validity of time-invariance [2] in nuclear beta decay can be tested if y is known, since the time-reversal violating amplitude is proportional to $y/(1+y^2)$. Furthermore, non-zero values of y imply the existence of isospin impurity in the nuclear states, which may arise either from the charge dependence of nuclear forces or from electromagnetic effects [3].

Since 1956, fourteen measurements [4–17] have been made on the asymmetry coefficient A for $2^+ \xrightarrow{\beta^+} 2^+ \xrightarrow{\gamma} 0^+$ of the $^{58}\text{Co} \to {}^{58}\text{Fe}$ decay with a view to obtaining y. However, such measurements by either polarized nuclei or β–γ circular polarization correlations in unpolarized nuclei are difficult, and different workers tend to obtain rather different values. Figure 1 gives fourteen independent measurements of y over the years. These experimentally-deduced values of y vary from roughly 0.05 to −0.36 with most values consistent with $y = 0$, particularly those from recent measuremens. The aim of this paper is to obtain a theoretical value of y and discusss the value so obtained in relation to time-reversal invariance.

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Calculation and Results

The Fermi nuclear matrix element M_F is related [1] to y through the relation

$$|M_F| = \left(\frac{2\,\text{ft (superallowed)}}{\text{ft (decay under study)}}\right)^{\frac{1}{2}} \frac{y}{(1+y^2)^{\frac{1}{2}}} \,. \qquad (1)$$

It has been shown [18] that, if a nucleus is deformed, calculations using shell-model wave functions are incorrect because of inhibitions due to the ΔK selection rule in β-decay. However, calculations [19–21] using the Nilsson model [22] with a one-body spheroidal Coulomb potential to obtain M_F seem to give reasonably agreement between theory and experiment, and we shall use the same approach.

We assume that the deformed nucleus ^{58}Co has the rotational band $K = 2$ and the deformed ^{58}Fe has $K = 0$ as shown in Fig. 2, where $|G\rangle$, $|P\rangle$, $|A\rangle$ and $|T_<\rangle$ are the ground state, the parent state, the analogue state and the anti-analogue state, respectively. By the K-selection rule $\Delta K \leqq 1$ for beta transitions, the beta matrix elements with $\Delta K = 2$ vanish. The experimentally observed decay is due to the admixture of other K bands to the $K = 2$ ground state of ^{58}Co and to the $K = 0$ excited state of ^{58}Fe. Assuming axially symmetric prolate deformation, in the initial state

$$\begin{aligned} |i\rangle &= |J=2, M, K=2, T=2, T_z=-2\rangle \\ &+ \bar{a}_0\,|J=2, M, K=0, T=2, T_z=-2\rangle \\ &+ \bar{\alpha}_2\,|J=2, M, K=2, T=3, T_z=-2\rangle \\ &+ \bar{a}_0\,\bar{\alpha}_0\,|J=2, M, K=2, T=3, T_z=-2\rangle \\ &+ \cdots \end{aligned} \qquad (2)$$

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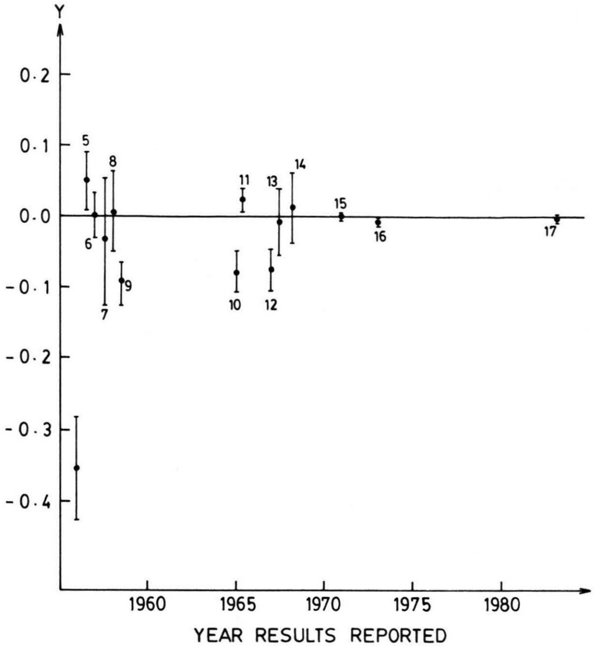

Fig. 1. Plot of all experimental values of the Fermi to Gamow-Teller mixing ratios y that have been reported. The numbers that label the data points refer to references.

and in the final state

$$\begin{aligned} |f\rangle = & |J=2, M, K=0, T=3, T_z=-3\rangle \\ & + a_2 |J=2, M, K=2, T=3, T_z=-3\rangle \\ & + a_0 |J=2, M, K=0, T=4, T_z=-3\rangle \\ & + a_2 \alpha_2 |J=2, M, K=2, T=4, T_z=-3\rangle \\ & + \cdots \end{aligned} \tag{3}$$

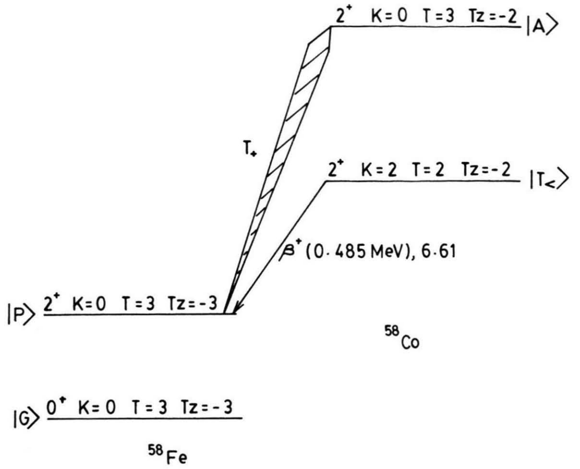

Fig. 2. Partial decay scheme of ^{58}Co.

with ΔE as the separation energy and V_c is the one-body spheroidal coulomb potential given by

$$\begin{aligned} V_c &= \frac{(Z-1)\,e^2}{R}\left\{\tfrac{3}{2} - \tfrac{1}{2}(r/R)^2\right\} + a(r/R)^2\,Y_{20} \quad \text{for } r<R, \\ V_c &= \frac{(Z-1)\,e^2}{r} + a(R/r)^3\,Y_{20} \quad \text{for } r>R, \end{aligned} \tag{7}$$

where R is the nuclear radius and a is related to the Bohr deformation parameter β by

$$a = \tfrac{3}{5}\,\beta(Z-1)\,\frac{e^2}{R}. \tag{8}$$

If we take [23, 24] $\beta \approx 0.1$, we obtain $\alpha_2 = 1.60 \times 10^{-3} \gg \bar{\alpha}_0$, which can be neglected.

The Gamow-Teller (GT) matrix element is calculated from the relation

$$M_{\rm GT}^2 = \frac{1}{2J+1} \sum_{\mu, M_i, M_f} |\langle J, M_f, K_f, T_f, T_{zf}|\, D_{\rm GT}(\mu)\, |J, M_i, K_i, T_i, T_{zi}\rangle|^2. \tag{9}$$

where $\bar{a}_0$ is the admixture amplitude of the $K=0$ in the initial state while a_2 that of the $K=2$ in the final state. $\bar{\alpha}_0, \bar{\alpha}_2, \ldots$ are isospin impurity amplitudes in the initial state and $\alpha_0, \alpha_2, \ldots$ are those in the final state.

The Fermi matrix element is

$$M_{\rm F} = \langle f\,|\,T_-\,|\,i\rangle = \sqrt{6}\,(\bar{a}_0\,\bar{\alpha}_0 + a_2\,\bar{\alpha}_2), \tag{4}$$

where

$$\bar{\alpha}_0 = -\frac{\langle J=2, M, K=0, T=2, T_z=-2|\,V_c\,|J=2, M, K=0, T=3, T_z=-2\rangle}{\Delta E}, \tag{5}$$

$$\bar{\alpha}_2 = -\frac{\langle J=2, M, K=2, T=2, T_z=-2|\,V_c\,|J=2, M, K=2, T=3, T_z=-2\rangle}{\Delta E}. \tag{6}$$

Calculation yields $|M_{\rm GT}| = 0.726\,a_2$. Using $M_{\rm GT} = C_{\rm v}\,M_{\rm F}/C_{\rm A}\,y$, and (1), we obtain $|M_{\rm GT}| = 0.03322$, from which $a_2 = 0.0457$.

This yields $M_{\rm F} = 1.79 \times 10^{-4}$, from which we deduce $y = 2.07 \times 10^{-4}$, and such a small value of y can be regarded as zero experimentally. Hence our result is consistent with time-reversal invariance. The vanishing value of y indicates that the Fermi contribution is strongly suppressed in this decay, not only due to ΔT selection rule but also due to ΔK selection rule.

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